TWO PARTICULAR SOLUTIONS OF THE PROBLEM OF MOTION OF A BODY WITH A FIXED POINT

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There are fourteen known exact particular solutions of the problem in question. They are all listed in [1], where it is noted that the equations of motion of a body are much simpler when one of the special coordinate axes coincides with the principal axis and when the gyrostatic moment is orthogonal to this axis. In this case the problem reduces to a system of four relatively simple differential equations in five variables related by an algebraic expression. This system admits of two exact solutions representable as segments of trigonometric series in some variable τ related to time by a differential expression.

1. Under the conditions $\lambda = \lambda_1 = \lambda_2 = 0$, $b_2 = 0$ Eqs. (1.1)-(1.4) of [1] are

$$x^{\cdot} = -z\,[(a_1 - a_2)y + bx], \qquad y^{\cdot} = z\,[(a - a_2)\,x + by] - \gamma_2$$
$$\gamma^{\cdot} = a_2 z\gamma_1 - (a_1 y + bx)\,\gamma_2, \qquad \gamma_1^{\cdot} = (ax + by)\gamma_2 - a_2 z\gamma$$
$$ax^2 + a_1 y^2 + a_2 z^2 + 2bxy - 2\,\gamma = 2E, \qquad x\gamma + y\gamma_1 + z\gamma_2 = k \tag{1.1}$$

Following [3], we introduce the variable τ,

$$d\tau = a_2 z dt$$

Setting $U(\tau) = \gamma_2 / a_2 z$, $h = 2E/a_2$ and referring the quantities a, a_1, b, k, γ, γ_1, γ_2 to a_2, we arrive at a system of equations describing the motion of a body in the case under consideration,

$$dx\,/\,d\tau = -(a_1 - 1)y - bx, \qquad dy\,/\,d\tau = (a - 1)x + by - U$$
$$d\gamma\,/\,d\tau = \gamma_1 - (a_1 y + bx)U, \qquad d\gamma_1\,/\,d\tau = -\gamma + (ax + by)U \tag{1.2}$$

$$x\gamma + y\gamma_1 + (h + 2\gamma - ax^2 - a_1 y^2 - 2\,bxy)\,U = k \tag{1.3}$$

We obtained the latter equation by eliminating z^2 from integrals (1.1). The variables z and γ_2 are given by Formulas

$$z^2 = h + 2\gamma - ax^2 - a_1 y^2 - 2\,bxy, \quad \gamma_2 = zU \tag{1.4}$$

The following integral is obtained:

$$\gamma^2 + \gamma_1^2 + \gamma_2^2 = \Gamma^2 \quad (\Gamma = mgr_c\,/\,a_2) \tag{1.5}$$

Here r_c is the distance from the fixed point to the center of mass of the body; mg is the weight of the body. Instead of (1.3) we shall henceforth make use of the equivalent (by virtue of (1.2)) relation

$$y\,d\gamma\,/\,d\tau - x\,d\gamma_1\,/\,d\tau + (2\,\gamma + h)\,U = k \tag{1.6}$$

2. Noting that system (1.2) is linear in x, y, γ, γ_1 for a given $U = U(\tau)$, we stipulate that the function U is of the form

$$U=\sum_{n=-2}^{2} U_n e^{in\nu\tau} \tag{2.1}$$

Eqs. (1.2) now define x, y, γ, γ_1 as functions of τ,

$$x=\sum_{n=-2}^{2} x_n e^{in\nu\tau},\quad y=\sum_{n=-2}^{2} y_n e^{in\nu\tau},\quad \gamma=\sum_{n=-4}^{4} \gamma_n e^{in\nu\tau},\quad \gamma_1=\sum_{n=-4}^{4} \gamma_n' e^{in\nu\tau} \tag{2.2}$$

Here x_n, y_n, γ_n, γ_n' are known functions of a, a_1, U_m, ν. Denoting by μ and c the expressions

$$\mu=(a-1)(a_1-1)-b^2,\ c=b^2-a(a_1-1) \tag{2.3}$$

we can write out the corresponding formulas

$$x_n=\frac{1-a_1}{n^2\nu^2-\mu}U_n,\quad y_n=\frac{b+i\nu n}{n^2\nu^2-\mu}U_n \qquad (n=0,\ \pm 1,\ \pm 2) \tag{2.4}$$

$$\gamma_n=\frac{1}{n^2\nu^2-1}\sum_{s=n-2}^{2}\frac{i\nu b(n-s)+(c+a_1\nu^2 ns)}{s^2\nu^2-\mu}U_sU_{n-s},\quad \gamma_{-n}=\bar{\gamma}_n \tag{2.5}$$

$$\gamma_n'=\frac{1}{n^2\nu^2-\mu}\sum_{s=n-2}^{2}\frac{b(\nu^2 ns-1)-i\nu(cn+a_1 s)}{s^2\nu^2-\mu}U_sU_{n-s},\quad \gamma_{-n}'=\overline{\gamma_n'} \quad (n=0,1,2,3,4)$$

In particular,

$$\gamma_4=\frac{2i\nu b-(c+8a_1\nu^2)}{(16\nu^2-1)(4\nu^2-\mu)}U_2^2,\quad \gamma_4'=\frac{b(8\nu^2-1)-2i\nu(a_1+2c)}{(16\nu^2-1)(4\nu^2-\mu)}U_2^2$$
$$\gamma_3=\frac{1}{9\nu^2-1}\left[\frac{i\nu b-(c+6a_1\nu^2)}{4\nu^2-\mu}+\frac{2i\nu b-(c+3a_1\nu^2)}{\nu^2-\mu}\right]U_1U_2 \tag{2.6}$$
$$\gamma_3'=\frac{1}{9\nu^2-1}\left[\frac{b(6\nu^2-1)-i\nu(2a_1+c)}{4\nu^2-\mu}+\frac{b(3\nu^2-1)-i\nu(a_1+3c)}{\nu^2-\mu}\right]U_1U_2$$

Substituting (2.1) and (2.2) into (1.6), we require that the resulting equation of the form

$$\sum_{n=-6}^{6} R_n e^{in\nu\tau}=k$$

where the R_n depend on x_m, y_m, γ_m, γ_m', U_m, h, ν, be an identity in τ. Since $R_{-n}=R_n$, it is enough to set $k=R_0$, $R_n=0$ ($n=1,\ldots,6$). The condition $R_6=0$ is of the form $2i\nu\gamma_4 y_2-2i\nu\gamma_4' x_2+\gamma_4 U_2=0$. Expanding it in accordance with Formulas (2.4) and (2.6), we find that

$$b=0,\ \ \nu^2=a(a-1)(a_1-1)/4(2a-a_1) \tag{2.7}$$

But for $b=0$ the equation $R_5=0$, i.e.

$$4i\nu\gamma_4 y_1+3i\nu\gamma_3 y_2-4i\nu\gamma_4' x_1-3i\nu\gamma_3' x_2+2\gamma_4 U_1+2\gamma_3 U_2=0,$$

is (by virtue of (2.4) and (2.6)) equivalent to

$$4\nu^6(346aa_1-180a_1^2-346a+173a_1)+\nu^4(a_1-1)(-710a^2a_1+216aa_1^2+710a^2+$$
$$+501aa_1-216a_1^3-821a+268a_1)+\nu^2(a_1-1)^2(a-1)(82a^2a_1-82a^2-82aa_1+$$
$$+136a-17a_1)-6a(a_1-1)^3(a-1)^2=0$$

On substituting ν^2 from (2.7) into this expression we obtain

$$a_1^3(16a^2-16a+4)+a_1^2a(-50a^2+33a-4)+$$
$$+a_1a^2(34a^2+17a-16)+a^3(-34a+16)=0$$

The latter equation has the three roots $a_1^{(1)}$, $a_1^{(2)}$, $a_1^{(3)}$. The values $a_1^{(1)}=a$ and $a_1^{(2)}=2a/(2a-1)$ yield a singularity in the denominators of Expressions (2.4) and (2.5) $4\nu^2-\mu=0$ for $a_1=a_1^{(1)}$ and $16\nu^2-1=0$ for $a_1=a_1^{(2)}$; solutions of the above class do not exist in this case.

For $a_1=a_1^{(3)}$ we obtain

$$a_1^{(3)} = \frac{a(17a-8)}{4(2a-1)}, \qquad \nu^2 = \frac{(1-a)(17a^2-16a+4)}{4a} \tag{2.8}$$

Since $\nu^2 > 0$ and since, moreover, the triangle inequalities for the moments of inertia, i.e.

$$\frac{1}{a_1} + \frac{1}{a} \geqslant 1, \qquad \frac{1}{a} + 1 \geqslant \frac{1}{a_1}, \qquad \frac{1}{a_1} + 1 \geqslant \frac{1}{a}$$

must be fulfilled, we find that a assumes values from the ranges

$$\frac{17-\sqrt{17}}{34} \leqslant a \leqslant \frac{\sqrt{273}-1}{34}, \qquad \frac{17+\sqrt{17}}{34} \leqslant a < 1 \tag{2.9}$$

3. Before investigating the remaining equations $R_n = 0$ ($n = 1,\ldots, 4$), it will be convenient to isolate the quantities U_m in the expressions for x_n, y_n, γ_n, γ_n'. We introduce X_m, Y_m, $\Gamma_{l,m}$, $\Gamma'_{l,m}$ in such a way that (3.1)

$$x_n = X_n U_n, \quad y_n = i\nu Y_n U_n, \quad \gamma_n = \sum_s \Gamma_{s,\,n-s} U_s U_{n-s}, \quad \gamma_n' = i\nu \sum_s \Gamma'_{s,\,n-s} U_s U_{n-s}$$

Comparing these equations with (2.4) and (2.5), we find that (3.2)

$$X_2 = X_{-2} = \frac{1-a_1}{4\nu^2-\mu}, \quad X_1 = X_{-1} = \frac{1-a_1}{\nu^2-\mu}, \quad X_0 = \frac{a_1-1}{\mu}$$

$$Y_2 = -Y_{-2} = \frac{2}{4\nu^2-\mu}, \quad Y_1 = -Y_{-1} = \frac{1}{\nu^2-\mu}, \quad Y_0 = 0$$

$$\Gamma_{2,2} = \Gamma_{-2,-2} = -\frac{c+8a_1\nu^2}{(16\nu^2-1)(4\nu^2-\mu)}$$

$$\Gamma_{1,2} = \Gamma_{-2,-1} = -\frac{1}{9\nu^2-1}\left(\frac{c+6a_1\nu^2}{4\nu^2-\mu} + \frac{c+3a_1\nu^2}{\nu^2-\mu}\right)$$

$$\Gamma_{0,2} = \Gamma_{-2,0} = \frac{1}{4\nu^2-1}\left(\frac{c}{\mu} - \frac{c+4a_1\nu^2}{4\nu^2-\mu}\right), \qquad \Gamma_{1,1} = \Gamma_{-1,-1} = -\frac{c+2a_1\nu^2}{(4\nu^2-1)(\nu^2-\mu)}$$

$$\Gamma_{-1,2} = \Gamma_{-2,1} = -\frac{1}{\nu^2-1}\left(\frac{c+2a_1\nu^2}{4\nu^2-\mu} + \frac{c-a_1\nu^2}{\nu^2-\mu}\right)$$

$$\Gamma_{0,1} = \Gamma_{-1,0} = \frac{1}{\nu^2-1}\left(\frac{c}{\mu} - \frac{c+a_1\nu^2}{\nu^2-\mu}\right)$$

$$\Gamma_{-2,2} = \frac{2c}{4\nu^2-\mu}, \quad \Gamma_{-1,1} = \frac{2c}{\nu^2-\mu}, \quad \Gamma'_{2,2} = -\Gamma'_{-2,-2} = -\frac{2(a_1+2c)}{(16\nu^2-1)(4\nu^2-\mu)}$$

$$\Gamma_{0,0} = -\frac{c}{\mu}, \quad \Gamma_{1,2}' = -\Gamma'_{-2,-1} = -\frac{1}{9\nu^2-1}\left(\frac{2a_1+3c}{4\nu^2-\mu} + \frac{a_1+3c}{\nu^2-\mu}\right)$$

$$\Gamma_{0,2}' = -\Gamma'_{-2,1} = \frac{2}{\nu^2-1}\left(\frac{c}{\mu} - \frac{a_1+c}{4\nu^2-\mu}\right), \quad \Gamma_{1,1}' = -\Gamma'_{-1,-1} = -\frac{a_1+2c}{(4\nu^2-1)(\nu^2-\mu)}$$

$$\Gamma'_{-1,2} = -\Gamma'_{-2,1} = \frac{-1}{1\nu^2-1}\left(\frac{2a_1+c}{4\nu^2-\mu} + \frac{c-a_1}{\nu^2-\mu}\right)$$

$$\Gamma_{0,1}' = -\Gamma'_{-1,0} = \frac{1}{\nu^2-1}\left(\frac{c}{\mu} - \frac{a_1+c}{\nu^2-\mu}\right), \qquad \Gamma'_{-2,2} = \Gamma'_{-1,1} = \Gamma_{0,0}' = 0$$

All of the quantities just introduced depend only on a by way of Formulas (2.3) and (2.8). The remaining equations $R_n = 0$ ($n = 1,\ldots, 4$) can be written as

$$\begin{gathered} \alpha_1 U_2 U_0 + \alpha_2 U_1^2 = 0, \;\; \beta_1 U_2^2 U_{-1} + \beta_2 U_2 U_1 U_0 + \beta_3 U_1^3 = 0 \\ \delta_1 U_2^2 U_{-2} + \delta_2 U_2 U_1 U_{-1} + \delta_3 U_2 U_0^2 + \delta_4 U_1^2 U_0 + hU_2 = 0 \\ \varepsilon_1 U_2 U_0 U_{-1} + \varepsilon_2 U_2 U_1 U_{-2} + \varepsilon_3 U_1^2 U_{-1} + \varepsilon_4 U_1 U_0^2 + hU_1 = 0 \end{gathered} \tag{3.3}$$

Here

$$\alpha_1 = 2\nu^2(-\Gamma_{0,2}Y_2 + 2\Gamma'_{2,2}X_0 + \Gamma'_{0,2}X_2) + 2(\Gamma_{2,2} + \Gamma_{0,2})$$

$$\alpha_2 = \nu^2(-3\Gamma_{1,2}Y_1 - 2\Gamma_{1,1}Y_2 + 3\Gamma'_{1,2}X_1 + 2\Gamma'_{1,1}X_2) + 2(\Gamma_{1,2} + \Gamma_{1,1})$$

$$\beta_1 = \nu^2(-4\Gamma_{2,2}Y_{-1} - \Gamma_{-1,2}Y_2 + 4\Gamma'_{2,2}X_{-1} + \Gamma'_{-1,2}X_2) + 2(\Gamma_{2,2} + \Gamma_{-1,2})$$

$$\beta_2 = \nu^2(-2\,\Gamma_{0,2}Y_1 - \Gamma_{0,1}\;Y_2 + 3\,\Gamma'_{1,2}X_0 + 2\Gamma'_{0,2}X_1 + \Gamma'_{0,1}X_2) + 2\,(\Gamma_{1,2} + \Gamma_{0,2} + \Gamma_{0,1})$$
$$\beta_3 = 2\nu^2(-\Gamma_{1,1}\,Y_1 + \Gamma'_{1,1}X_1) + 2\Gamma_{1,1}$$

$$\delta_1 = 4\nu^2(-\Gamma_{2,2}Y_{-2} + \Gamma'_{2,2}X_{-2}) + 2\,(\Gamma_{2,2} + \Gamma_{-2,2}) \tag{3.4}$$
$$\delta_2 = \nu^2(-3\Gamma_{1,2}\,Y_{-1} - \Gamma_{-1,2}\;Y_1' + 3\Gamma'_{1,2}X_{-1} + \Gamma'_{-1,2}X_1) + 2\,(\Gamma_{1,2} + \Gamma_{-1,2} + \Gamma_{-1,1})$$
$$\delta_3 = 2\nu^2\Gamma'_{0,2}X_0 + 2\,(\Gamma_{0,2} + \Gamma_{0,0}), \qquad \delta_4 = \nu^2(-\Gamma_{0,1}Y_1 + 2\Gamma'_{1,1}\;X_0 + \Gamma'_{0,1}X_1) + 2\,(\Gamma_{1,1} + \Gamma_{0,1})$$

$$\varepsilon_1 = \nu^2(-2\Gamma_{0,2}Y_{-1} + \Gamma_{-1,0}Y_2 + 2\Gamma'_{0,2}X_{-1} + \Gamma'_{-1,2}X_0 - \Gamma'_{-1,0}X_2) + 2\,(\Gamma_{0,2} + \Gamma_{-1,2} + \Gamma_{-1,0})$$
$$\varepsilon_2 = \nu^2(-3\Gamma_{1,2}Y_{-2} + \Gamma_{-2,1}Y_2 + 3\Gamma'_{1,2}X_{-2} - \Gamma''_{-2,1}X_2) + 2\,(\Gamma_{1,2} + \Gamma_{-2,2} + \Gamma_{-2,1})$$
$$\varepsilon_3 = 2\nu^2(-\Gamma_{1,1}Y_{-1} + \Gamma'_{1,1}X_{-1}) + 2\,(\Gamma_{1,1} + \Gamma_{-1,1}), \quad \varepsilon_4 = \nu^2\Gamma_{0,1}X_0 + 2\,(\Gamma'_{0,1} + \Gamma_{0,0})$$

4. The first three equations of (3.3) define the squares of the absolute values of U_n,

$$U_{-2}U_2 = |U_{-2}|^2 = |U_2|^2 = \frac{(\alpha_2\beta_2 - \alpha_1\beta_3)^2}{\alpha_2^2\beta_1^2}\,U_0^2$$

$$U_{-1}U_1 = |U_{-1}|^2 = |U_1|^2 = \frac{\alpha_1(\alpha_2\beta_2 - \alpha_1\beta_3)}{\alpha_2^2\beta_1}\,U_0^2 \tag{4.1}$$

$$U_0^2 = \frac{\alpha_2^2\beta_2^2 h}{\alpha_2\beta_1^2(\alpha_1\delta_4 - \alpha_2\delta_3) - \alpha_1\beta_1\delta_2(\alpha_2\beta_2 - \alpha_1\beta_3) - \delta_1(\alpha_2\beta_2 - \alpha_1\beta_3)^2}$$

The quantities U_n are generally complex,

$$U_n = |U_n| \exp i\varphi_n, \qquad U_{-n} = \overline{U_n} \qquad (n = 1,\ 2)$$

and, as is evident from the first equation of (3.3), their arguments ϕ_n are related by Expressions

$$\varphi_2 = \pi + \arg\alpha_2 - \arg\alpha_1 + 2\,\varphi_1, \qquad \varphi_{-2} = \pi - \arg\alpha_2 + \arg\alpha_1 - 2\varphi_1$$

Making use of the fact that system (1.2) is self-contained, we incorporate the constant ϕ_1 into $\nu\tau$, which enables us to regard the U_n as real functions of a. Hence,

$$U = \sum_{n=-2}^{2} |U_n| \exp i\,(n\nu\tau + \varphi_n) = U_0 + 2U_1\cos\nu\tau + 2U_2\cos 2\nu\tau$$

and we can assume that $U_0 > 0$, $U_1 > 0$.

Thus, the solution of Eqs. (1.2), (1.6) is of the form (cf. (2.2), (3.1))

$$U = \sqrt{h}\sum_{n=0}^{2}(U_n)\cos n\nu\tau, \quad x = \sqrt{h}\sum_{n=0}^{2}(x_n)\cos n\nu\tau, \quad y = \sqrt{h}\sum_{n=1}^{2}(y_n)\sin n\nu\tau$$

$$\gamma = h\sum_{n=0}^{4}(\gamma_n)\cos n\nu\tau, \quad \gamma_1 = h\sum_{n=1}^{4}(\gamma_n')\sin n\nu\tau$$

$$(U_n) = \frac{2U_n}{\sqrt{h}}, \quad (x_n) = \frac{2x_n}{\sqrt{h}}, \quad (y_n) = \frac{2y_n}{i\sqrt{h}} \qquad (n = -1,\ -2)$$

$$(\gamma_n) = \frac{2\gamma_n}{h}, \quad (\gamma_n') = \frac{2\gamma_n'}{ih} \qquad (n = -1,\ -2,\ -3,\ -4)$$

$$(U_0) = \frac{U_0}{\sqrt{h}}, \qquad (x_0) = \frac{x_0}{\sqrt{h}}, \qquad (\gamma_0) = \frac{\gamma_0}{h}$$

The variables z^2 and $\gamma_2^{\,2}$ can be determined from relations (1.4),

$$z^2 = h\sum_{n=0}^{4}(z_n)\cos n\nu\tau, \qquad \gamma_2^{\,2} = h^2\sum_{n=0}^{8}(\gamma_n'')\cos n\nu\tau$$

From the condition of realness of z we infer that $\nu\tau$ varies in the range

$$\Psi_1 + 2m\pi \leqslant \nu\tau \leqslant \Psi_2 + 2\,m\pi \qquad (m = 0,\ \pm 1,\ \pm 2,\ \ldots)$$

Integral (1.5) yields $\varkappa^2 h^2 = \Gamma^2$ i.e. the relationship between h and Γ.
The dependence on t can be determined from the relation $d\tau = a_2 z dt$,

$$t' = a_2 t = \frac{1}{\sqrt{h}} \int_{\tau_1}^{\tau} \left[\sum_{n=0}^{4} (z_n) \cos n\nu\sigma \right]^{-1/2} d\sigma$$

To complete construction of the solution, let us write out the condition which a must satisfy. Substituting (4.1) into the last equation of (3.3), we obtain

$$(\varepsilon_2 - \delta_1)(\alpha_2\beta_2 - \alpha_1\beta_3)^2 + \beta_1(\alpha_1\varepsilon_3 - \alpha_2\varepsilon_1 + \alpha_1\delta_2)(\alpha_2\beta_2 - \alpha_1\beta_3) + \\ + \alpha_2\beta_1^2(\alpha_2\varepsilon_4 + \alpha_1\delta_4 - \alpha_2\delta_3) = 0$$

which has the three roots $a^{(1)}$, $a^{(2)}$, $a^{(3)}$ in ranges (2.9). The value $a^{(3)} = 0.4$ must be rejected, since it yields $\Gamma = 0$. We have thus obtained two particular solutions of equations (1.2), (1.3). These solutions are exact, since the expressions for (x_n), (y_n), (γ_n), (γ_n'), (z_n), (γ_n'') which depend on $a^{(1)}$ and $a^{(2)}$ are known (cf. (2.3), (2.8), (3.2), (3.4), (4.1), (3.1)).

5. Let us write out these solutions, taking as our $a^{(1)}$ and $a^{(2)}$ their approximate values obtained numerically.

The first solution: $a^{(1)} = 0.41190$, $\nu = 0.32385$

$$U = \sqrt{h}\,(2.3910 + 1.5836 \cos \nu\tau - 0.7942 \cos 2\nu\tau)$$

$$x = -\sqrt{h}\,(4.0656 + 4.7056 \cos \nu\tau + 1.8994 \cos 2\nu\tau)$$

$$y = \sqrt{h}\,(3.6551 \sin \nu\tau + 2.9508 \sin 2\nu\tau)$$

$$z^2 = -h\,(2.8001 + 4.6960 \cos \nu\tau + 2.7124 \cos 2\nu\tau + \\ + 0.9920 \cos 3\nu\tau + 0.1731 \cos 4\nu\tau)\,10$$

$$\gamma = -h\,(0.5228 + 1.0615 \cos \nu\tau + 1.0048 \cos 2\nu\tau + 0.6263 \cos 3\nu\tau + 0.1763 \cos 4\nu\tau)\,10$$

$$\gamma_1 = h\,(1.0742 \sin \nu\tau + 1.2310 \sin 2\nu\tau + 0.6601 \sin 3\nu\tau + 0.1601 \sin 4\nu\tau)\,10$$

$$\gamma_2^2 = h^3\,(-3.1183 - 5.1569 \cos \nu\tau - 2.7854 \cos 2\nu\tau - 0.7584 \cos 3\nu\tau + 0.1127 \cos 4\nu\tau + \\ + 0.1680 \cos 5\nu\tau + 0.0416 \cos 6\nu\tau - 0.0048 \cos 7\nu\tau - 0.0027 \cos 8\nu\tau)\,10^2$$

$$.026092\, h^2 = \Gamma^2, \quad 2.7431 + 2m\pi \leqslant \nu\tau \leqslant 3.5401 + 2m\pi \quad (m = 0, \pm 1, \pm 2, \ldots)$$

The second solution: $a^{(2)} = 0.70819$, $\nu = 0.35086$

$$U = \sqrt{h}\,(2.5242 + 0.9663 \cos \nu\tau + 0.4598 \cos 2\nu\tau)\,10^{-1}$$

$$x = -\sqrt{h}\,(8.6501 + 2.0852 \cos \nu\tau + 0.4701 \cos 2\nu\tau)\,10^{-1}$$

$$y = -\sqrt{h}\,(1.0197 \sin \nu\tau + 0.4597 \sin 2\nu\tau)\,10^{-1}$$

$$z^2 = h\,(1.1811 - 4.6670 \cos \nu\tau - 1.5776 \cos 2\nu\tau - 0.5912 \cos 3\nu\tau - 0.0264 \cos 4\nu\tau)\,10^{-1}$$

$$\gamma = -h\,(1.6253 + 0.9811 \cos \nu\tau + 0.4685 \cos 2\nu\tau + 0.3012 \cos 3\nu\tau + 0.0184 \cos 4\nu\tau)\,10^{-1}$$

$$\gamma_1 = h\,(-0.9572 \sin \nu\tau + 0.4482 \sin 2\nu\tau + 2.3858 \sin 3\nu\tau + 0.0764 \sin 4\nu\tau)\,10^{-2}$$

$$\gamma_2^2 = h^2\,(0.6549 + 3.8038 \cos \nu\tau + 2.2812 \cos 2\nu\tau + 1.4602 \cos 3\nu\tau + 0.4868 \cos 4\nu\tau + \\ + 0.1492 \cos 5\nu\tau + 0.0252 \cos 6\nu\tau + 0.0037 \cos 7\nu\tau + 0.0001 \cos 8\nu\tau)\,10^{-2}$$

$$).026574 h^2 = \Gamma^2, \quad 1.0099 + 2m\pi \leqslant \nu\tau \leqslant 5.2733 + 2m\pi \quad (m = 0, \pm 1, \pm 2, \ldots)$$

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